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Minimalism about Truth
Lecture #2: The Proper Formulation

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How To Be a Minimalist about Truth?

The following is an *instance of the T-schema*.

(1) 'Snow is white' is true iff snow is white.

The T-schema appears to be correct for any other sentence that you may substitute for 'snow is white'.

Except maybe for sentences like 'this sentence is false', see below

If you are not a minimalist, you may want to use your theory of truth (whatever it is) to explain why all/most instances of the T-schema are correct.

In fact, many might say that you *have* to

If you are a minimalist, you claim that the T-schema is 'all there is' to truth. The *utility* of the truth predicate is that it figures in constructions like:

(2) Whatever you ask me, I will give a true answer.
(*intensional* use)

(3) I don't know what Thomas said, but I am sure what he said is true.
(*opaque reference* use)

(4) Given any sentence, either it is true or its negation is true.
(*universal* use)

Minimalism: If you have *other means* to phrase such sentences, you do not need a truth predicate.

Non-minimalists might contend that we need a theory of truth even if the truth-predicate is eliminable from our language.

So here is a **challenge** for the minimalist: to **state the minimal theory** you cannot introduce such linguistic constructions that would make the truth predicate superflous—lest your account of truth is self-undermining.

This seems to be close to a refutation of minimalism: if the minimalist states her theory of truth, this seems to be some sort of *universal use*.

- To say that 'all instances of the T-schema are true' is presupposing a notion of truth and would hence be circular.
- To paraphrase this without appealing to truth seems to *explain universal uses without appealing to truth* and would hence undermine the minimalist project.

So we are in a bit of a pickle.

Today's plan is to state the **proper formulation** of minimalism, avoiding circularity and self-undermining.

There are other questions we will touch on: is minimalism a theory of truth, or a theory of the linguistic item 'true'? Does it answer the question 'what is Truth?'

Axioms and Axiom Schemas

This is a bit of an excursion, but hopefully helpful

This is an axiom of Peano Arithmetic. (An instance of Induction.)

(5) $(\mathsf{odd}(0) \land \forall n.(\mathsf{odd}(n) \to \mathsf{odd}(n'))) \to \forall n.\mathsf{odd}(n).$

But *this* is **not** an axiom of Peano Arithmetic.

(6) $\forall \varphi \forall \bar{y}.(\varphi(0, \bar{y}) \land \forall n.(\varphi(n, \bar{y}) \to \varphi(n', \bar{y}))) \to \forall n.\varphi(n, \bar{y}).$

Quantification over formulae is not possible in first-order logic.

Thus, we phrase Peano Arithmetic as having infinitely many axioms. A formula is one of the induction axioms of Peano Arithmetic if it can be obtained by substituting something for φ in (7).

You may notice that the number theorist never says anything like "a number is...". Do these infinitely many axioms make for an 'implicit' definition of number?

(7) $\forall \bar{y}.(\varphi(0, \bar{y}) \land \forall n.(\varphi(n, \bar{y}) \to \varphi(n', \bar{y}))) \to \forall n.\varphi(n, \bar{y}).$

Substitutional Quantification

Mathematicians are then wont to use a metalanguage device that philosophers call *substitutional quantification* and say an English sentence like 'every substitution of a formula for φ in (7) yields an arithmetical truth.'.

But note that such substitutional quantification is **very** different from our usual quantifiers that say something like 'all objects satisfy P'. Substitutional quantification does not say 'all formulae satisfy P'. For example, compare:

On the internet, people sometimes say "this!" as an agreement. That might be interesting. Colloquial English seems to know "Yeah, what he said." Make of this what you will.

(8) For any formula, if it is well-formed, then...
(usual quantification)

(9) For any formula, if it, then...
(substitutional quantification)

Substitutional quantification is also called "quantification into sentence position", as 'it' in (9) appears in the syntactic position of a sentence.

In any case, such quantifiers do not seem to occur in natural language. Sentences like (9) sound wrong.

We could use math to introduce such a construction, but then we wouldn't need Truth anymore. So we cannot help ourselves to substitutional quantification if we want to be minimalists.

Between these two options, the Minimalist contends that her theory is just *simpler* and preferable on Occam's Razor-like grounds. See below.

Quantifying over Propositions

To, say that we cannot help ourselves to substitutional quantification does not mean that we cannot quantify over propositions. To to so merely requires us to treat propositions as objects. Let's take for granted that **propositions are objects** and that we can conceptualise propositions without having a prior account of truth (lest we be circular).

Certainly, the concept of a proposition has a philosophical pedigree. Many people apply this concept without worrying too much. The Minimalist, at this point, need not say much more, since she *does not care* what propositions *are*.

So we may **allow ourselves to quantify over propositions**—they are objects after all.

But quantifying over propositions does not mean we can quantify into 'sentence position'. This is still bad:

(10) $\forall x.\mathsf{proposition}(x) \rightarrow x.$

This is just not how object-level quantification works. But this is okay:

(11) $\forall x.\mathsf{proposition}(x) \rightarrow \mathsf{abstract}(x).$

Let's also suppose that we have a concept of **meaning**.

But not the truth-conditional conception, as this would again be circular.

And let's suppose that **sentences** stand in some sort of relation to propositions that we may denote with $\langle . \rangle$.

We don't care whether the relation is 'denoting', 'expressing' or something else. That's up to a theory of meaning.

(12) 'Snow is white' denotes/expresses/means/… ⟨snow is white⟩.

Now, the following is an axiom of the Minimal Theory of Truth (MT).

(13) ⟨⟨snow is white⟩ is true iff snow is white⟩

This is a proposition that states what is required for the proposition ⟨snow is white⟩ to be true. This is what we want.

Asking here whether this axiom is *true* is misguided: this presupposes a truth-conditional notion of propositions, which we already acknowledged to be incompatible with minimalism.

The MT has infinitely many axioms. We cannot state them all (we are finite). Even with quantification over propositions we cannot state the MT finitely.

One attempt would be this:

(14) $\forall x.\mathsf{proposition}(x) \rightarrow \langle \mathsf{true}(x) \leftrightarrow x \rangle.$

This is quantifying into sentence position. What about this attempt?

(15) $\forall x.\mathsf{proposition}(x) \rightarrow \mathsf{true}(x) \leftrightarrow x.$

This is *also* quantifying into sentence position.

How To Be a Minimalist

But what we can do is this:

(16) $\forall x.\mathsf{proposition}(x) \rightarrow \mathsf{axiom\text{-}of\text{-}the\text{-}MT}(\langle \mathsf{true}(x) \leftrightarrow x \rangle).$

Technically this needs one more assumption: propositions have a **structure** that we can manipulate.

For example, given two propositions we can form their conjunction.

So let's define a function E that maps a proposition x to its corresponding MT-axiom.

(17) E(⟨snow is white⟩) = ⟨⟨snow is white⟩ is true iff snow is white⟩.

And then this is what the minimalist can tell you about their theory of truth.

(18) $\forall x.\mathsf{axiom\text{-}of\text{-}the\text{-}MT}(x) \leftrightarrow \exists y.\mathsf{proposition}(y) \wedge x = E(y).$

So we have a means of identifying axioms of the MT: for every object I can tell you whether it is such an axiom.

One minor technicality: there is one fact about truth that does not seem to be explained here—namely that only propositions can be true. 'Julius Caesar is true' is infelicitous and it seems incumbent on a theory of truth to explain it. So the MT contains one more axiom:

Horwich thinks this is not a big deal. Is it?

(19) $\forall x.\mathsf{true}(x) \rightarrow \mathsf{proposition}(x)$.

In what Sense is this a Theory of Truth?

Henri Poincare once said that 'a collection of facts is no more a science than a heap of stones is a house.'. Not just any collection of facts about a certain subject matter is a *theory* of that subject matter.

Not even if you would list *every* fact about a matter would this amount to a *theory*.

The Minimalist holds that the MT is a theory in that it is *fundamental to truth*, *maximally simple* and *fully explanatory*. That is, the MT explains *all* facts about truth and *no simpler* collection of facts about truth could do so.

But the MT isn't obviously simple. We struggled to get here. The MT cannot be fully stated and it is infinite.

The Minimalist rebuttal is this: any theory of truth must at least derive all the axioms of the MT.

- In this sense, the MT is *fundamental*.
- Any other theory that derives the infinitely many axioms is (most likely?) also infinite, and hence not simpler to state.

Substitutional Quantification, Redux

But wait, what about substitutional quantification? That seems to be a device that would allow us to phrase something finite from which to derive the MT.

But to do so, you need to give a finite theory of substitutional quantification.

Perhaps you'd give a rule of inference: (let $\tilde{\forall}$ be the substitutional universal).

(20) from $\tilde{\forall}p(\ldots p \ldots)$ infer $(\ldots q \ldots)$.

But that is not a theory of substitutional quantification, because you need to state (20) for every q.

To make that point clear. Suppose you have $\tilde{\forall}p(\ldots p \ldots)$ and want to infer $(\ldots$ snow is white$\ldots)$. You cannot infer this with (20). You'd need this.

(21) from $\tilde{\forall}p(\ldots p \ldots)$ infer $(\ldots$ snow is white$\ldots)$.

Of course you would want to say that (20) is true for all q. But to say this is *itself* a substitutional quantification. So nothing has been gained.

The problem is that you cannot explain universality without appealing to universality. Remember Achilles and the Tortoise.

Achilles fails to explain to the Tortoise how *modus ponens* works because he keeps appealing to more instances of *modus ponens* ("if you have 'if p then q' and 'p', then you have 'q' ").

The same game can be played for universals ("if 'all x are P' and you have **any** x, then P(x)").

Further Worries

Account of what?

Is the MT an account of *truth* or of the *word* 'true'?

The defender of MT claims that it is the former. For the *concept* of truth occurs in certain beliefs about the world and the MT explains what these beliefs *are*. Beliefs like the following:

Some minimalists, however, are contend with giving a theory "only" about the word 'true'.

(22) Whatever you ask Thomas, he will give a true answer.

(23) I don't know what Thomas said, but what he said is true.

(24) All propositions are true or false.

Compare with a belief like this

(25) Electrons have positive charge.

The theories of *electron* and *charge* explain what it is to believe (25). They are *not* just theories of the words 'electron' and 'charge'.

These are not *deflationary* theories of course.

Finish the sentence: "Truth is…"

The minimalist refuses. To be able to finish such sentences is an unreasonable high bar for something to count as a "theory".

Consider the Frege–Russell theory of definite description, with which everybody seems to be happy.

Note the quotes!

(26) 'The F is G' means 'there is an unique F and it is G'.

The MT is on equal ground here, using it we can say (for any 'p').

Note the quotes!

(27) 'It is true that p' means 'p'.

Whatever problem there could be with having to say this for all individual 'p' equally applies to the Frege–Russell theory that goes for all individual 'F'.

Liars!

What about the following axiom of the MT.

(28) $\langle\langle$this sentence is false$\rangle$ is true iff this sentence is false.$\rangle$

By our definition of the MT, this is an axiom. But it is obviously contradictory.

Begrudgingly, we have to say that this cannot be an axiom of the MT. So the "final" MT will look like this.

Other minimalists might say that this *is* an axiom of MT, but that the paradox is sorted out elsewhere.

(29) $\forall x.\text{MT-axiom}(x) \leftrightarrow \exists y.\text{unparadoxic}(y) \wedge \text{proposition}(y) \wedge x = E(y)$.

What precisely the 'unparadoxic' condition amounts to is not (yet?) known.

In any case, as all the 'good' axioms of the MT should be derived by *any* theory of truth (also a non-minmal one), whereas the Liar axioms should not be derived — this is everybody's problem.

More recent papers (notably, Bacon 2015; Murzi & Rossi 2019) indicate that the strategy of sorting certain 'paradoxic' propositions out of the MT is a hopeless affair. I see that differently.

I tend to think that restricting the MT for the Liar somewhat undermines the point about 'simplicity', so this challenge is bigger than Horwich makes it seem.